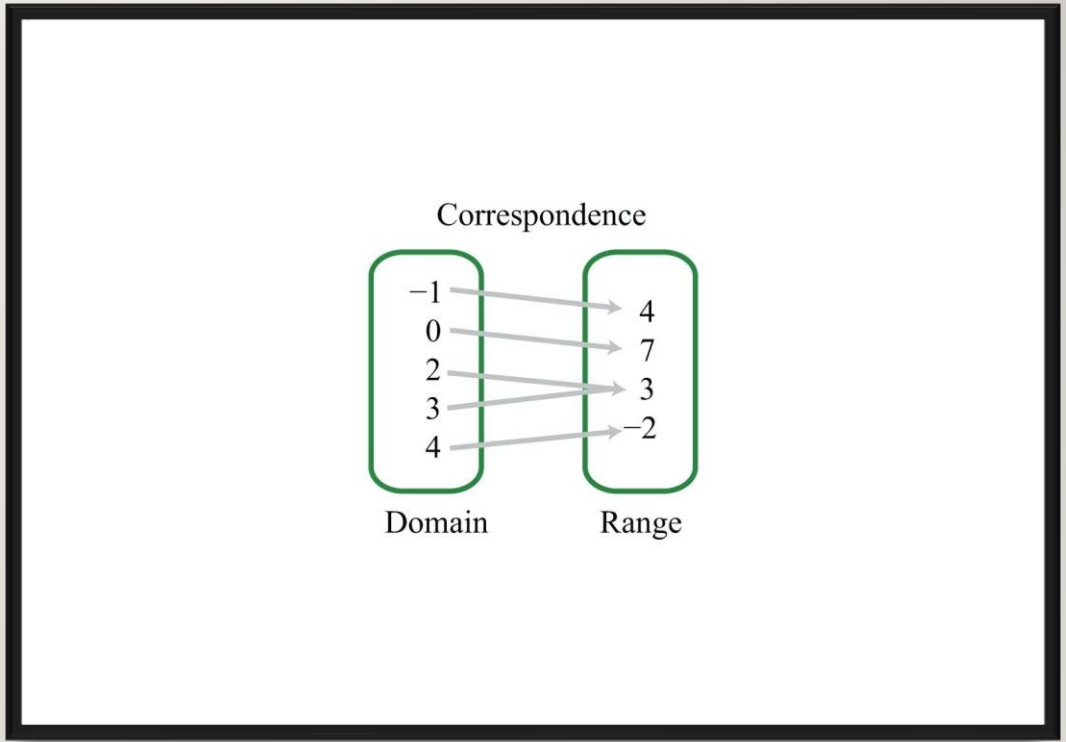
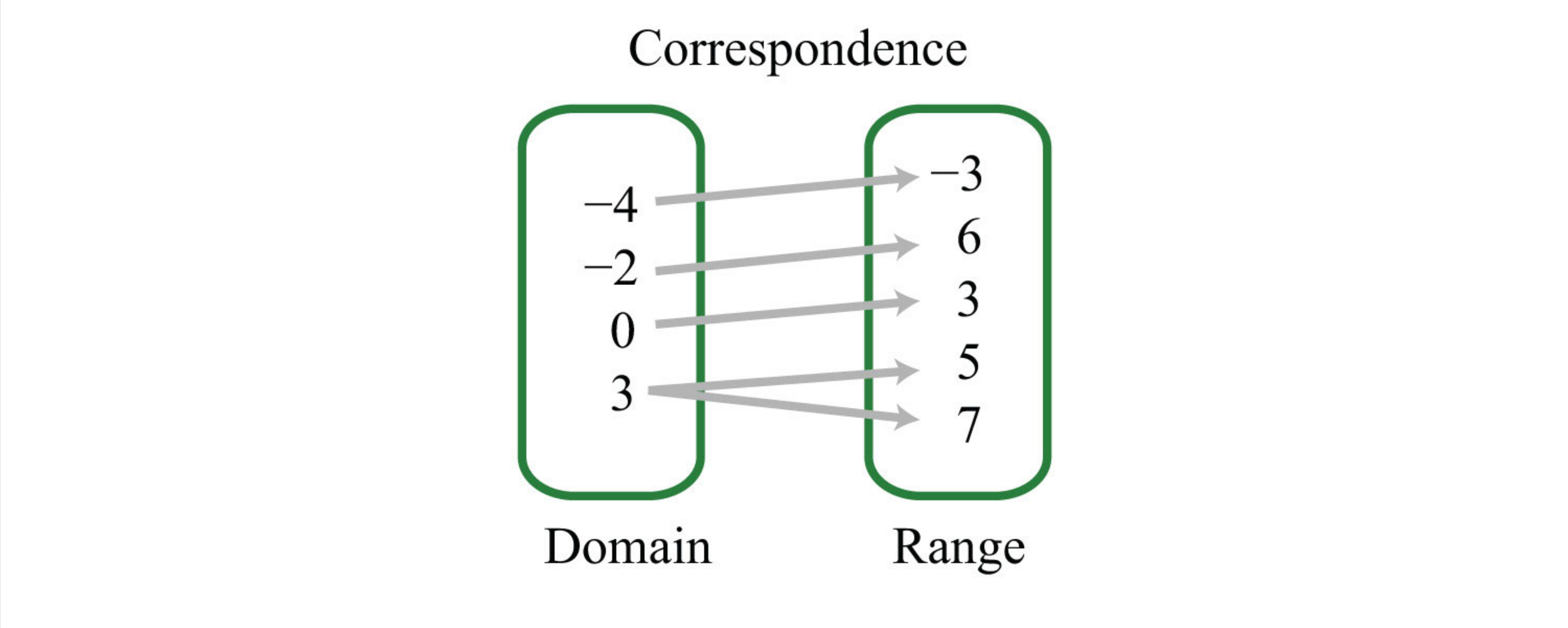
INTRO TO FUNCTIONS

WHAT IS A FUNCTION?

- Function is a "relation" between the sets that associates one input to a single output.
- The input values are the "domain" of a function
- The output values are the "range" of a function

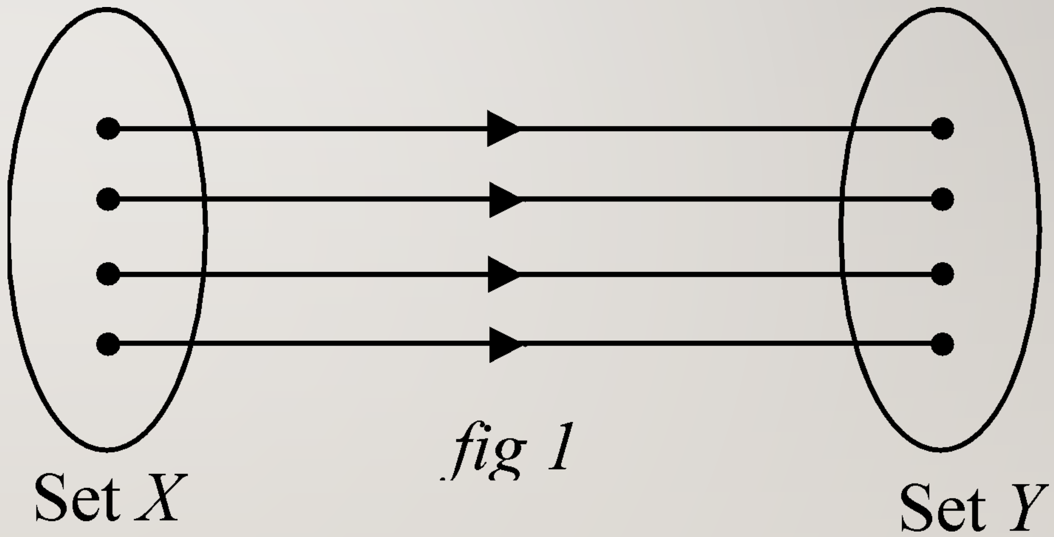


IS THIS A FUNCTION?

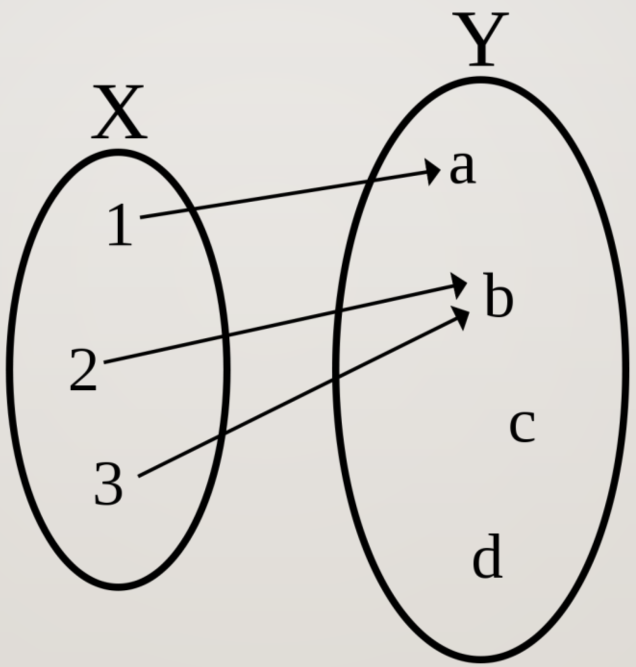


ONE-TO-ONE FUNCTION

- It is a FUNCTION
- Each input has a different output



IS THIS A ONE-TO-ONE FUNCTION?



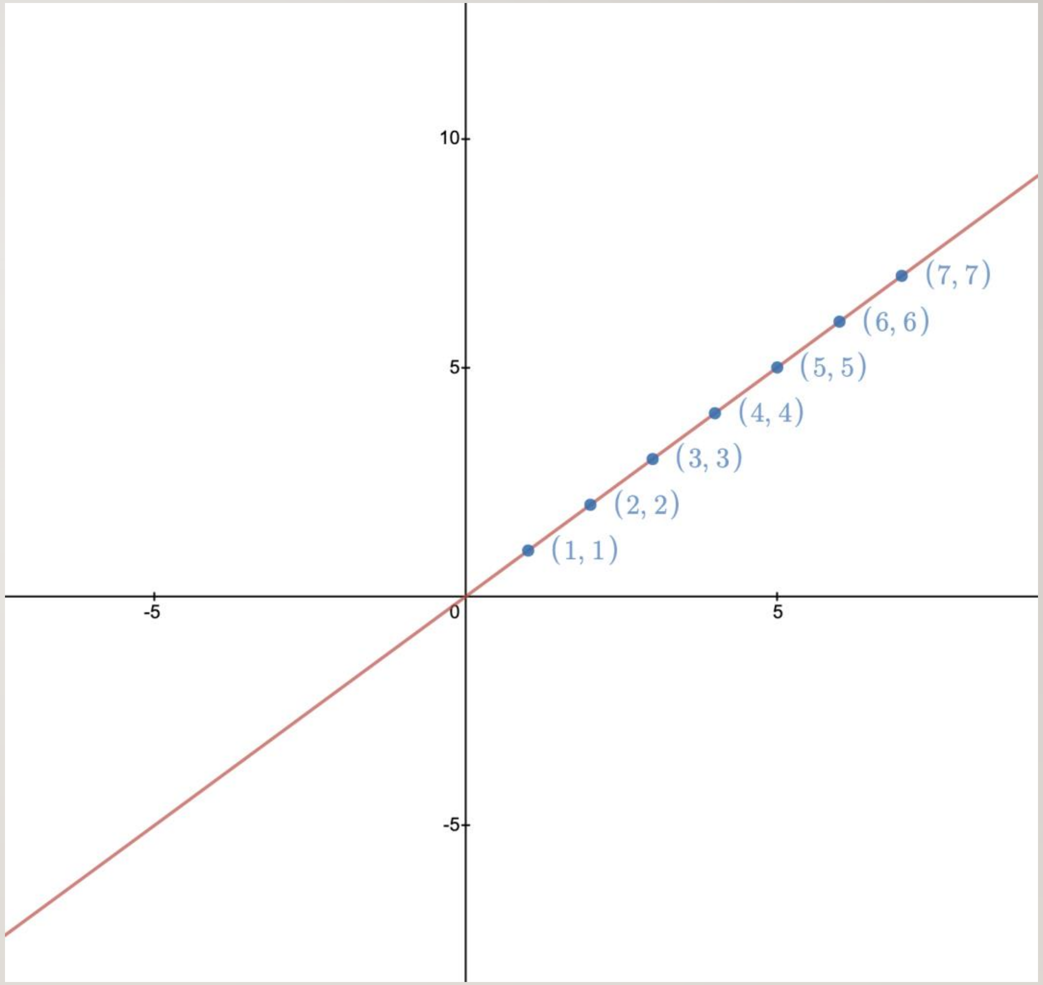
PLOTTING FUNCTIONS

$$y = x$$

x	y
1	1
2	2
3	3
4	4
5	5
6	6
7	7

$y = x$

- Passes "vertical line test"
  - It's a function
- Passes "horizontal line test"
  - It's a one-to-one function



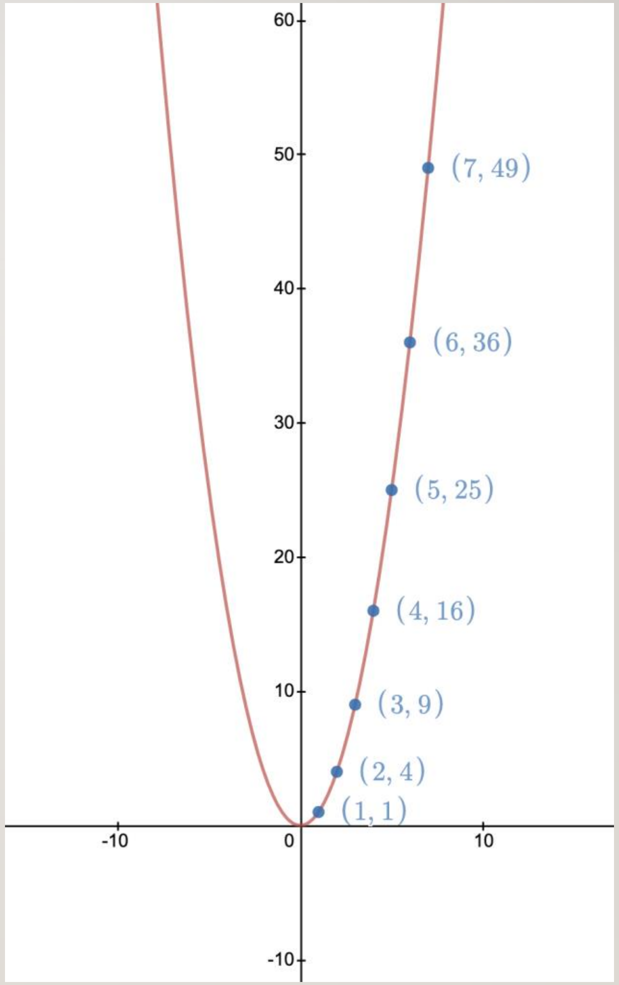
PLOTTING FUNCTIONS

$$y = x^2$$

x	y
1	2
2	4
3	9
4	16
5	25
6	36
7	49

$$y = x^2$$

- Passes "vertical line test"
  - It's a function
- DOESN'T pass "horizontal line test"
  - It's NOT a one-to-one



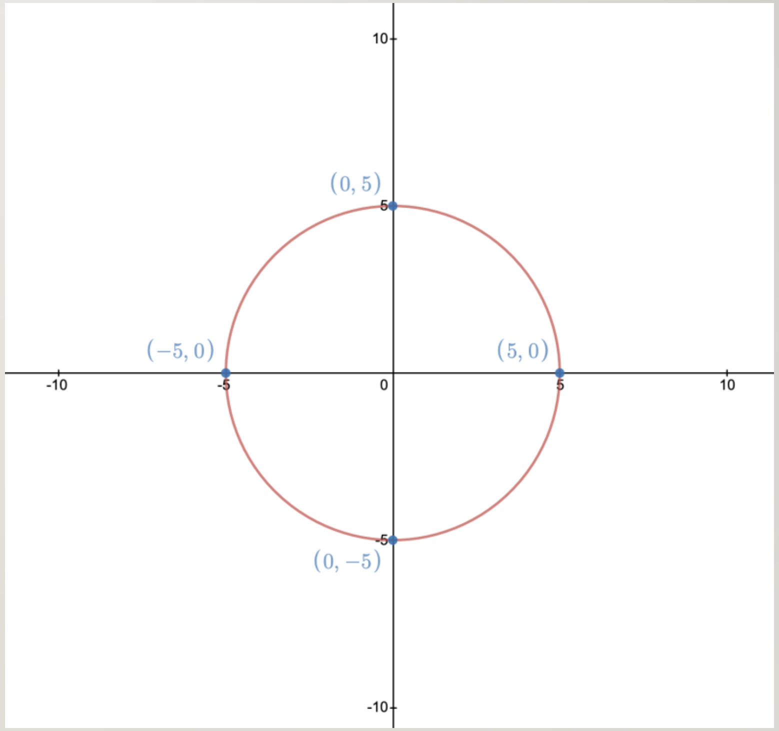
PLOTTING FUNCTIONS

$$x^2 + y^2 = 25$$

x	y
0	5
0	-5
-5	0
5	0

$x^2 + y^2 = 25$

- DOESN'T pass "vertical line test"
  - It's NOT a function
- DOESN'T pass "horizontal line test"
  - It's NOT a one-to-one

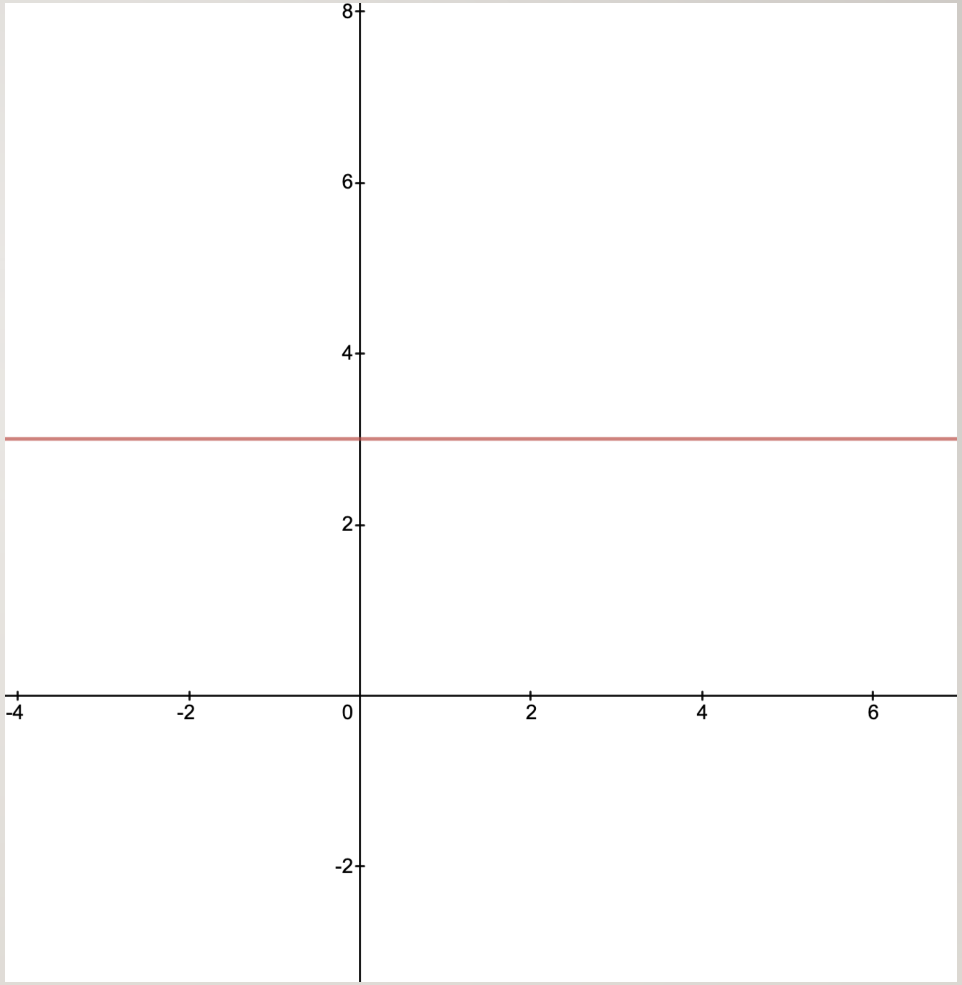


LET'S PRACTICE!

$$y = 3$$

Function?

One-to-one?

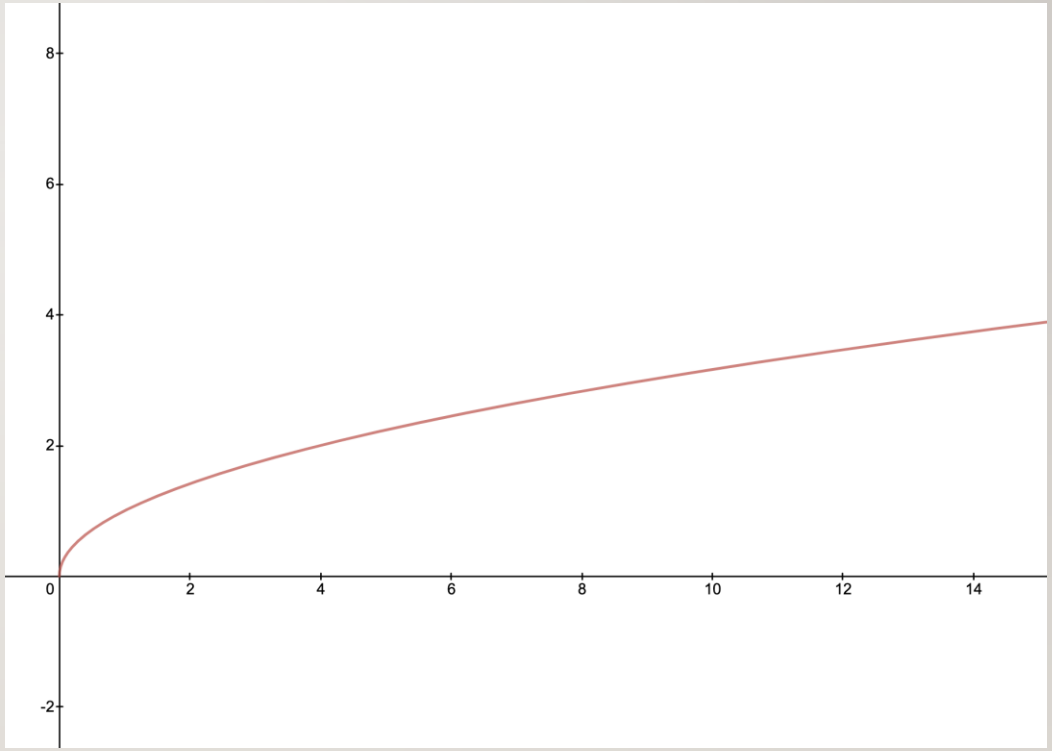


LET'S PRACTICE!

$$y = \sqrt{x}$$

Function?

One-to-one?

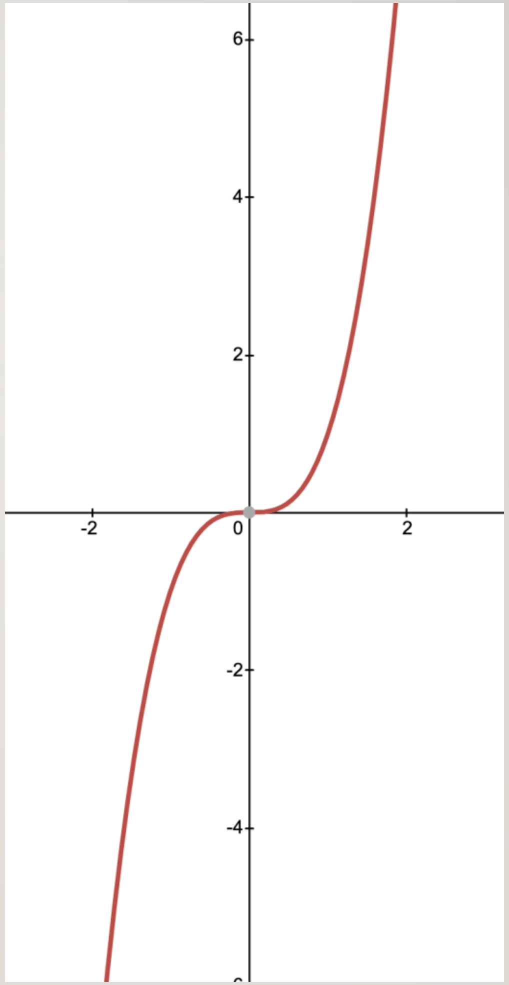


LET'S PRACTICE!

$$y = x^3$$

Function?

One-to-one?

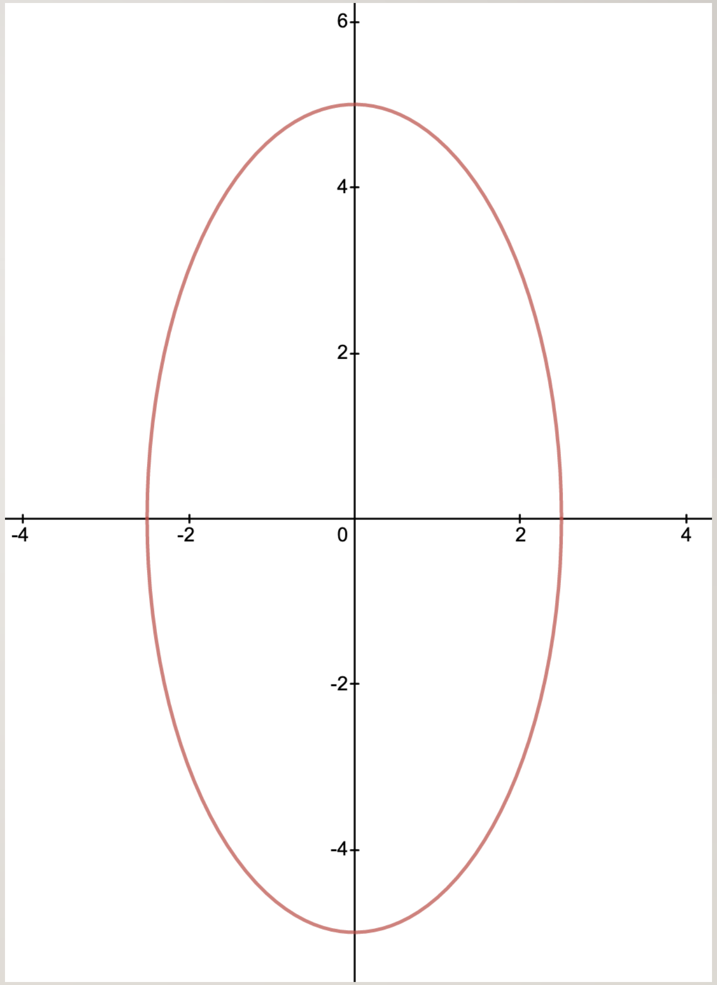


LET'S PRACTICE!

$$4x^2 + y^2 = 25$$

Function?

One-to-one?

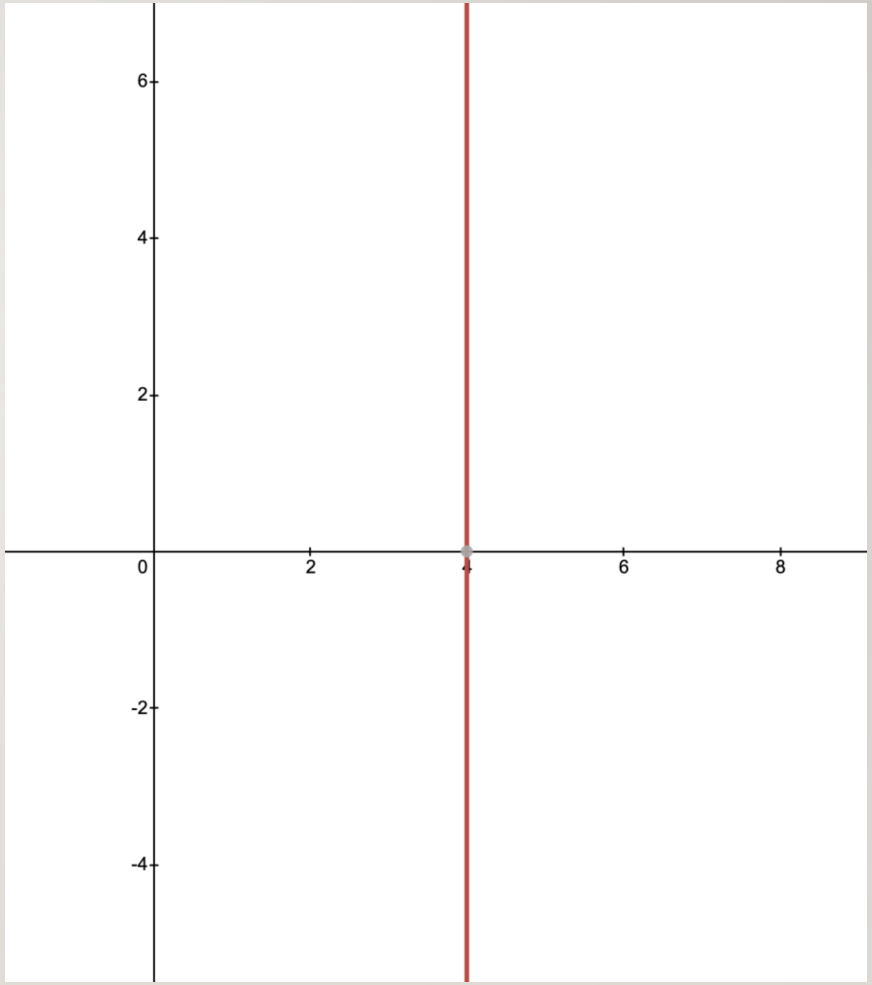


LET'S PRACTICE!

$$x = 4$$

Function?

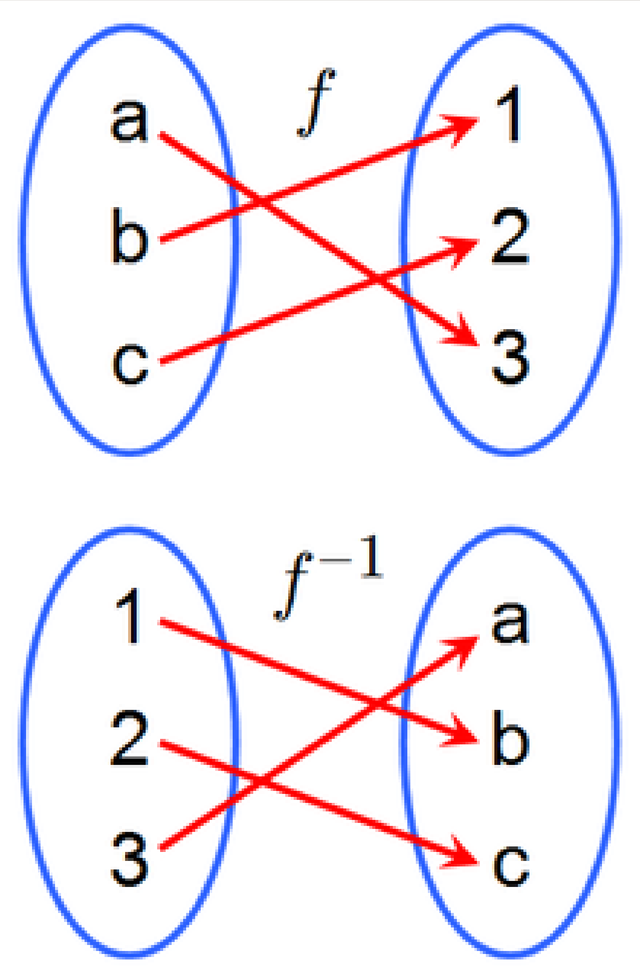
One-to-one?



INVERSE FUNCTIONS

WHAT IS AN INVERSE FUNCTION?

- Inverse function f^{-1} is a "reverse" of the original function f
  - f has an input x and output y
  - f^{-1} has an input y and output x
- The domain of f becomes the range and the range of f becomes the domain for f^{-1}



FINDING INVERSE FUNCTIONS

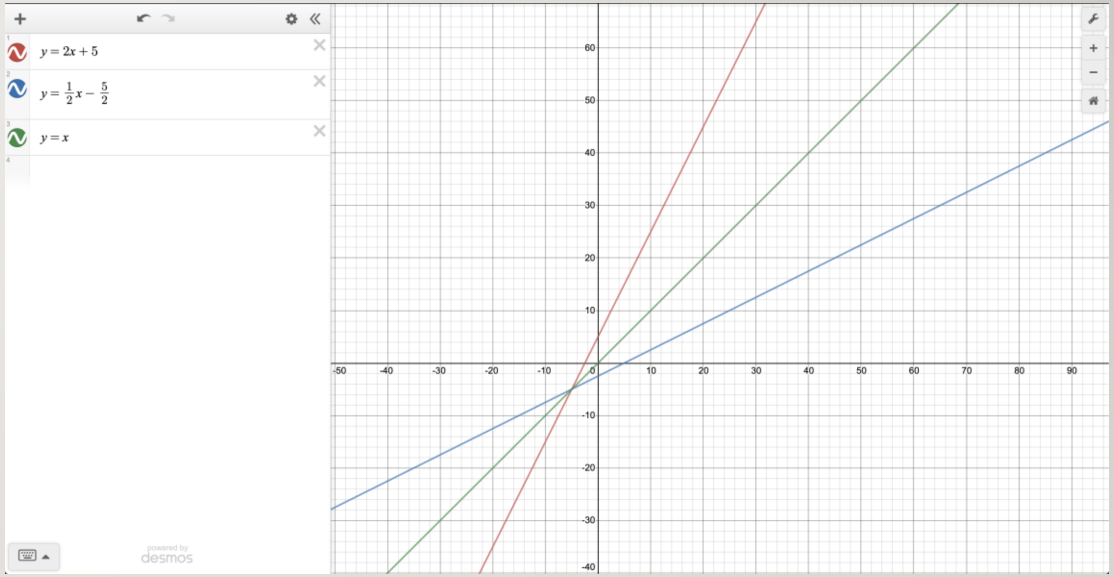
- Switch x and y
- Solve for y
- Replace y with $f^{-1}(x)$

FINDING INVERSE FUNCTIONS

- $y = 2x + 5$
- $x = 2y + 5$
- $x - 5 = 2y$
- $y = \frac{1}{2}x - \frac{5}{2}$
- $f^{-1}(x) = \frac{1}{2}x - \frac{5}{2}$

FINDING INVERSE FUNCTIONS

- Can also reflect the original function over $y = x$



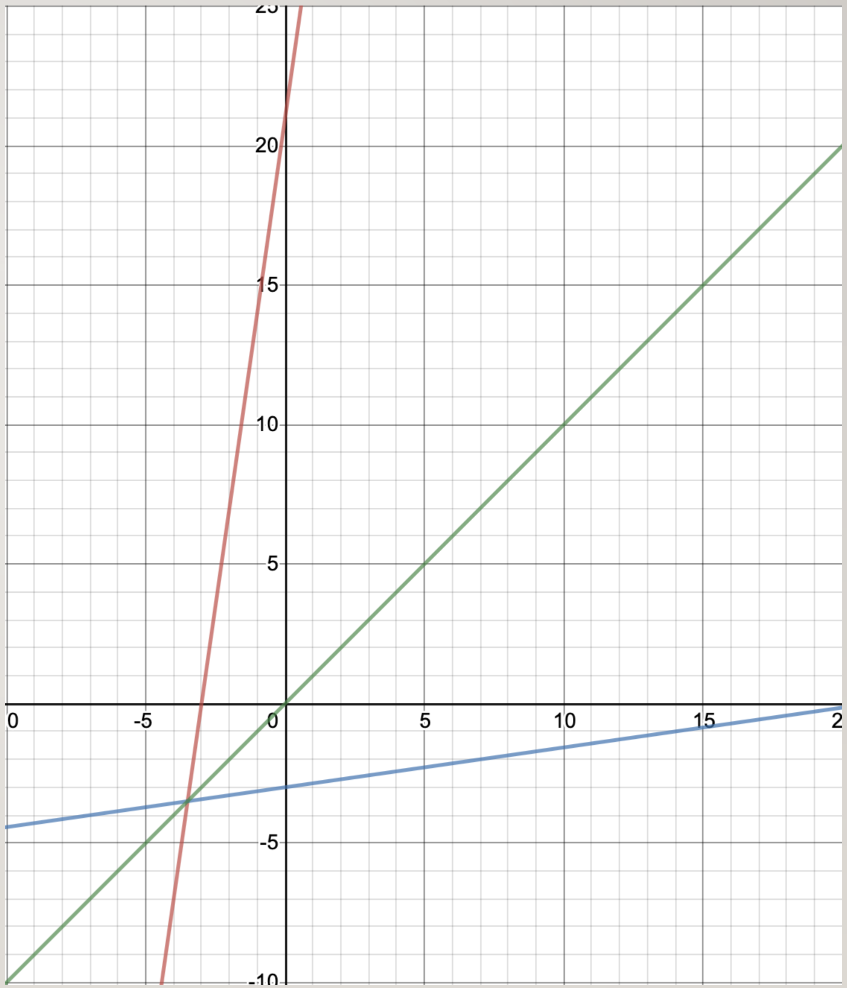
LET'S PRACTICE!

Find the inverse of the followings:

- $y = 7x + 21$
- $y = \frac{1}{4}x - 9$
- $y = e^x$
- $y = 3^x + 8$
- $y = x^2$

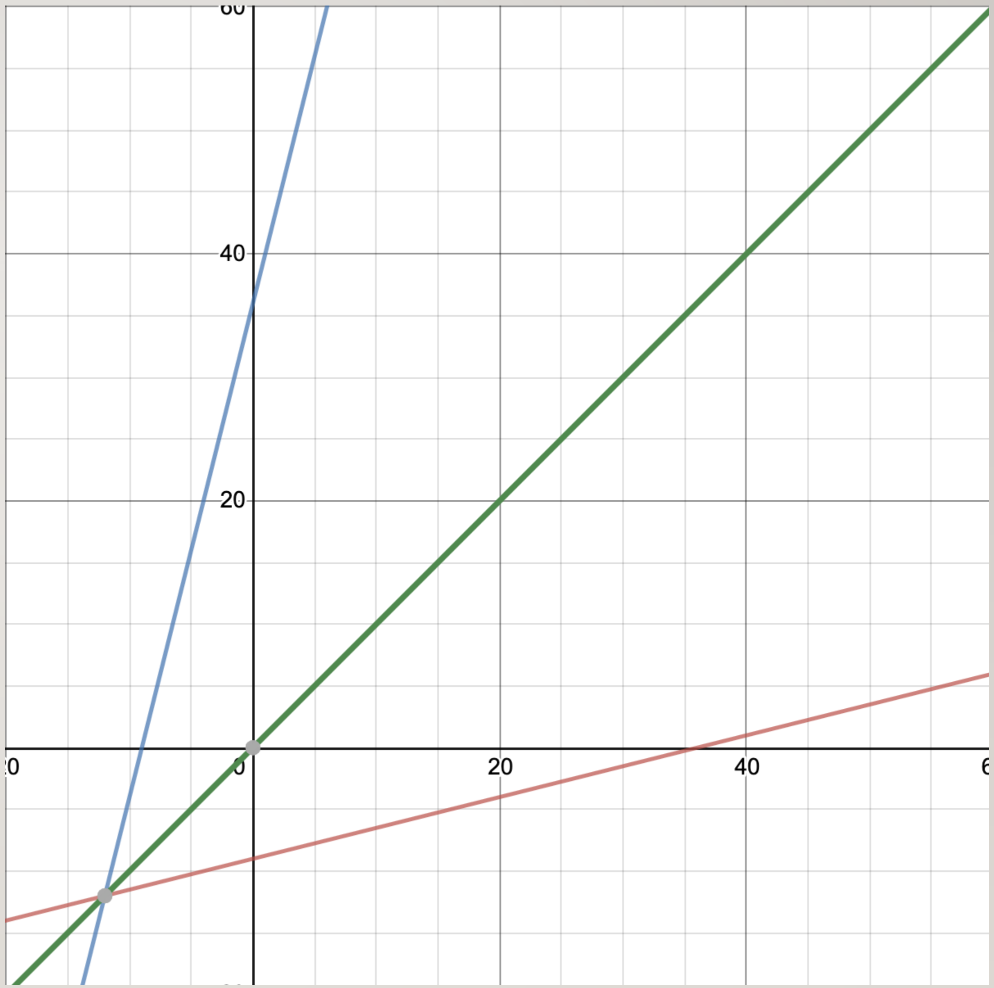
LET'S PRACTICE!

- $y = 7x + 21$
- $x = 7y + 21$
- $x - 21 = 7y$
- $y = \frac{1}{7}x - 3$
- $f^{-1}(x) = \frac{1}{7}x - 3$



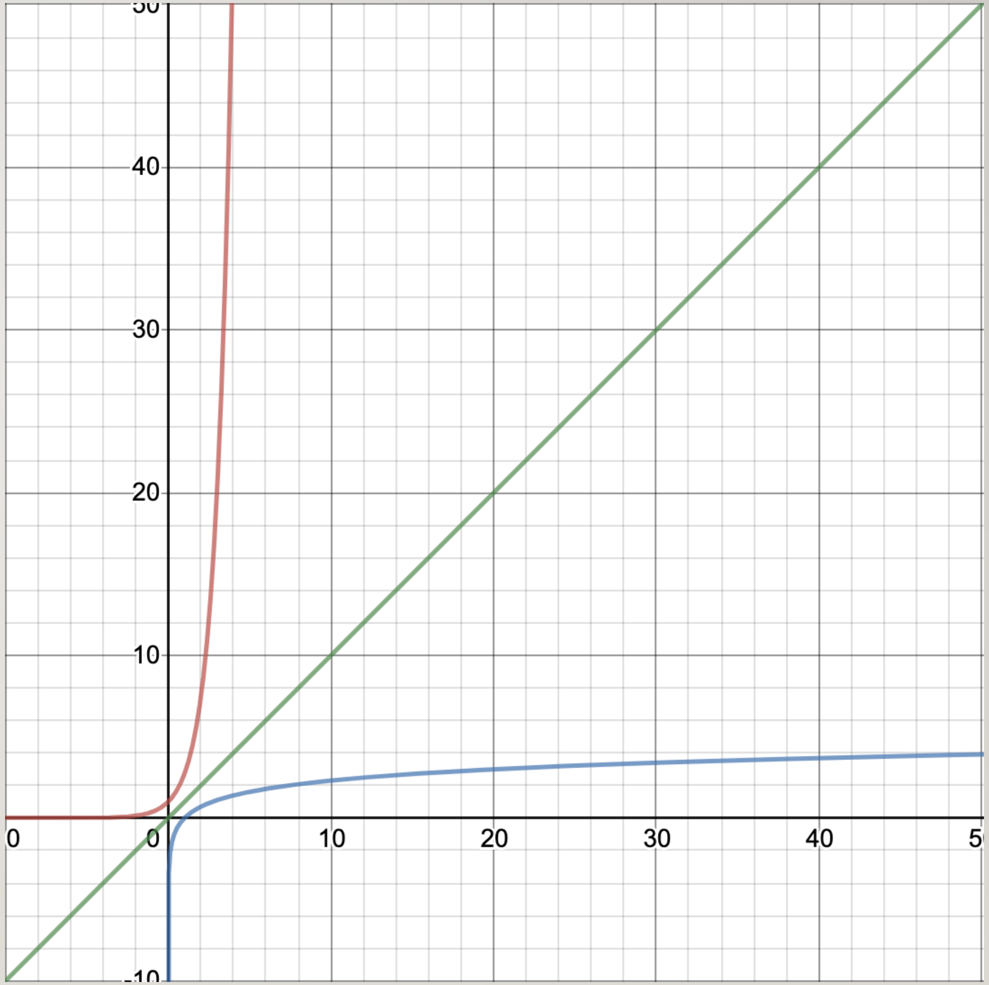
LET'S PRACTICE!

- $y = \frac{1}{4}x - 9$
- $x = \frac{1}{4}y - 9$
- $x + 9 = \frac{1}{4}y$
- $y = 4x + 36$
- $f^{-1}(x) = 4x + 36$



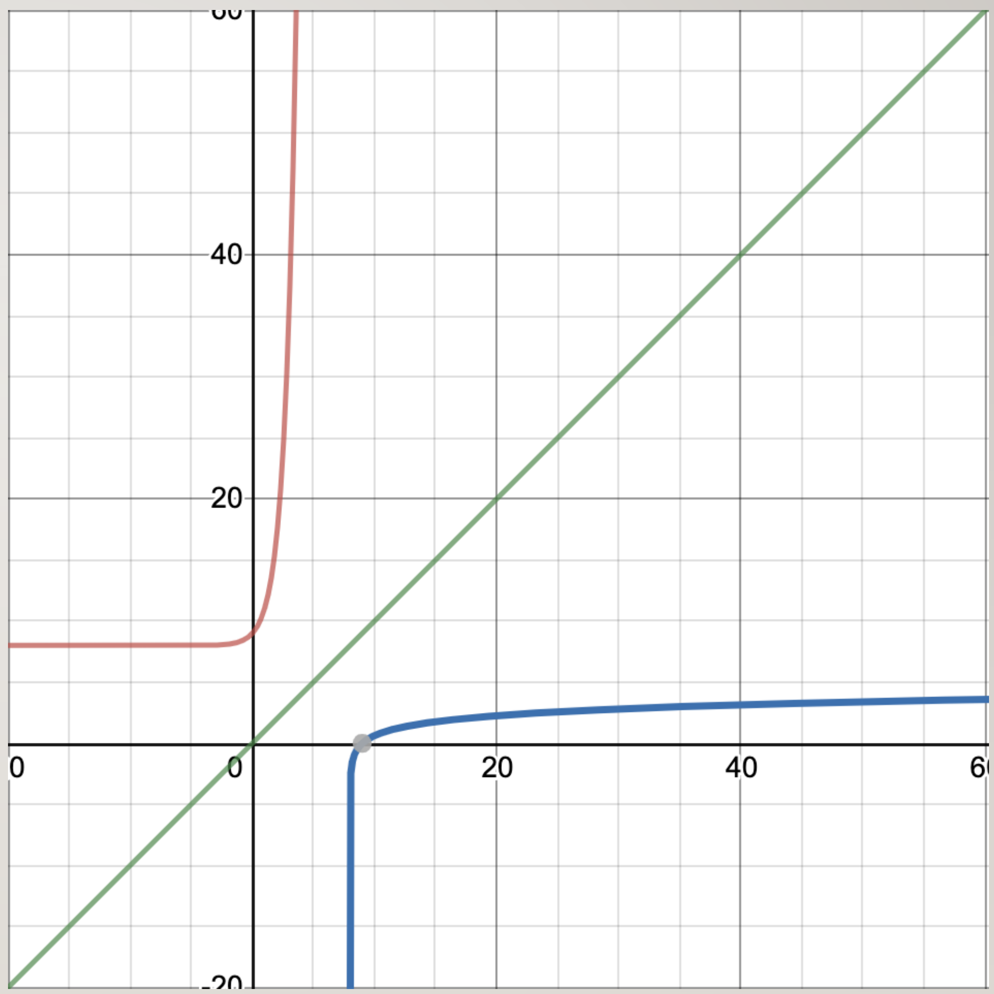
LET'S PRACTICE!

- $y = e^x$
- $x = e^y$
- $y = \ln x$
- $f^{-1}(x) = \ln x$



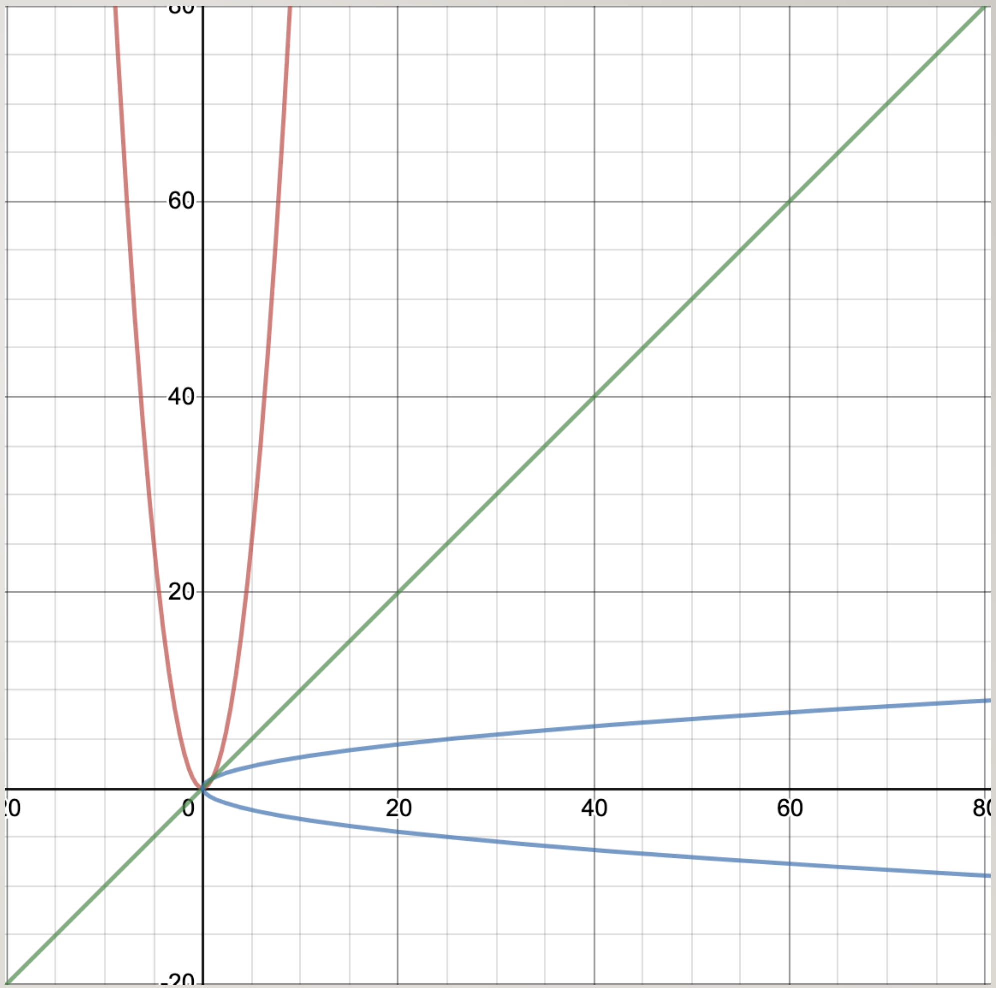
LET'S PRACTICE!

- $y = 3^x + 8$
- $x = 3^y + 8$
- $x - 8 = 3^y$
- $y = \log_3(x - 8)$
- $f^{-1}(x) = \log_3(x - 8)$



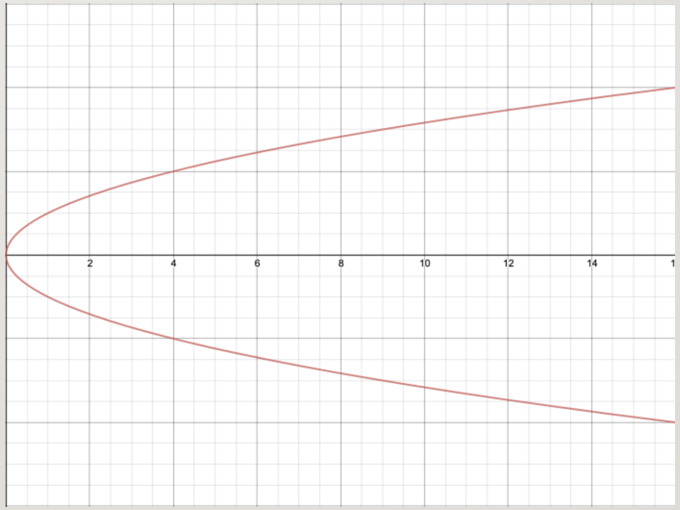
LET'S PRACTICE!

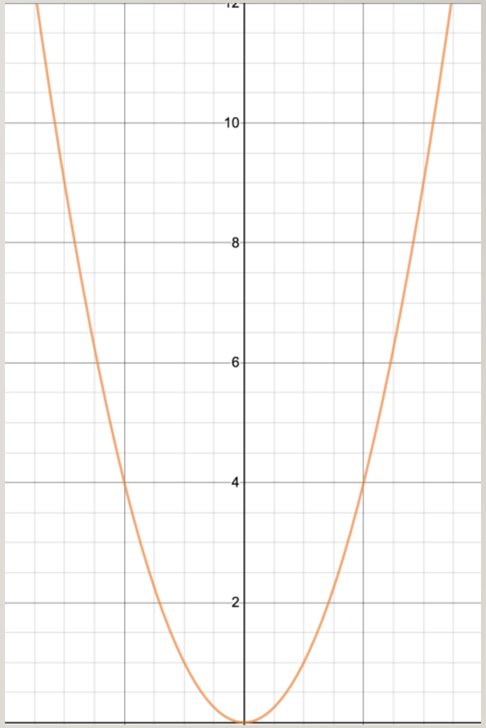
- $y = x^2$
- $x = y^2$
- $y = \pm\sqrt{x}$
- $f^{-1}(x) = \pm\sqrt{x}$



IS INVERSE A FUNCTION?

- $f^{-1}(x) = \pm\sqrt{x}$
  - This is not a function because it doesn't pass vertical line test
- $f(x) = x^2$
  - This is not one-to-one because it doesn't pass horizontal line test





CONDITIONS FOR THE INVERSE TO BE A FUNCTION

- The domain and range is switching. So, for its inverse to be a function(passes a vertical line test), the original function has to be one-to-one (passes a horizontal line test)

LET'S PRACTICE!

Determine if the inverses of the followings are functions:

- $y = 4x^3$
- $4x^2 + y^2 = 100$
- $y = x$
- $y = e^x$
- $y = x^6$

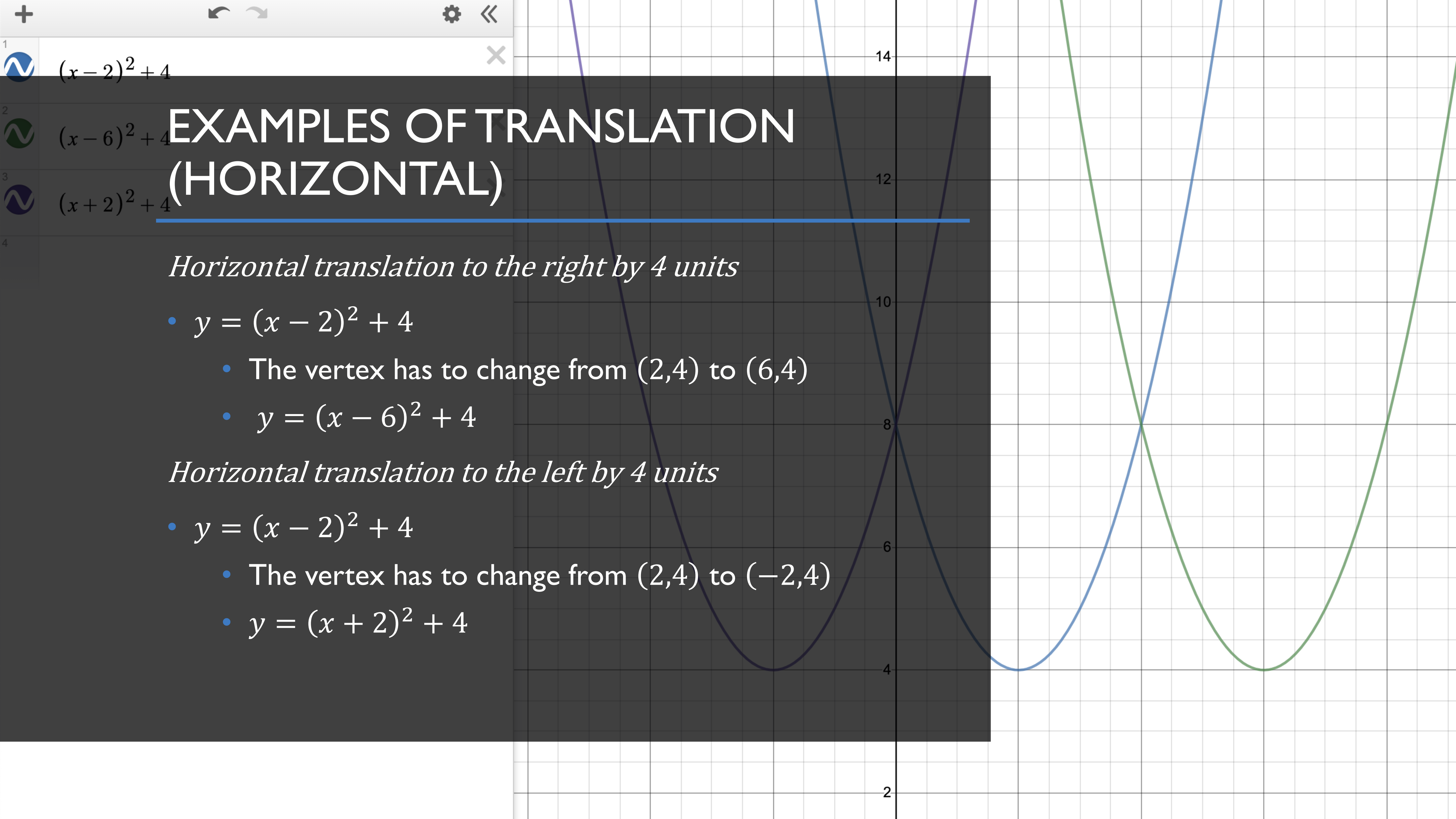
TRANSFORMATION OF FUNCTIONS

TYPES OF TRANSFORMATIONS

- Translation
  - The curve is shifted
- Reflection
  - The curve is flipped
- Dilation
  - The curve gets smaller/bigger

QUADRATIC FUNCTIONS

- Standard form
  - $y = ax^2 + bx + c$
- Vertex form
  - $y = a(x-h)^2 + k$
    - $(h, k) \Rightarrow vertex$
- Vertex form is better to identify the shape of the function, therefore the transformation of the function.



EXAMPLES OF TRANSLATION (HORIZONTAL)

Horizontal translation to the right by 4 units

- $y = (x-2)^2 + 4$
  - The vertex has to change from $(2,4)$ to $(6,4)$
  - $y = (x-6)^2 + 4$

Horizontal translation to the left by 4 units

- $y = (x-2)^2 + 4$
  - The vertex has to change from $(2,4)$ to $(-2,4)$
  - $y = (x+2)^2 + 4$

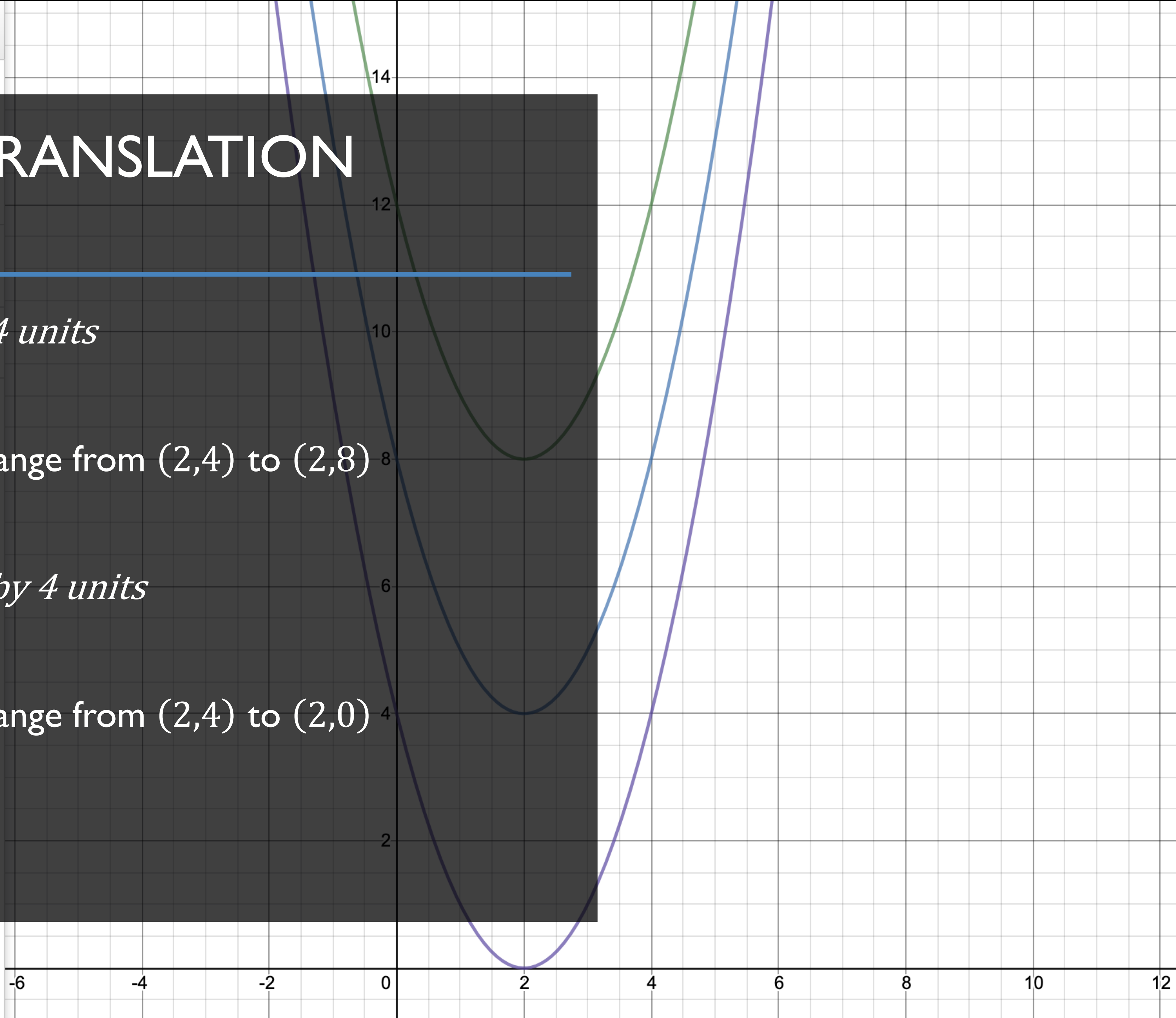
EXAMPLES OF TRANSLATION (VERTICAL)

Vertical translation up by 4 units

- $y = (x-2)^2 + 4$
  - The vertex has to change from (2,4) to (2,8)
  - $y = (x-2)^2 + 8$

Vertical translation down by 4 units

- $y = (x-2)^2 + 4$
  - The vertex has to change from (2,4) to (2,0)
  - $y = (x-2)^2$



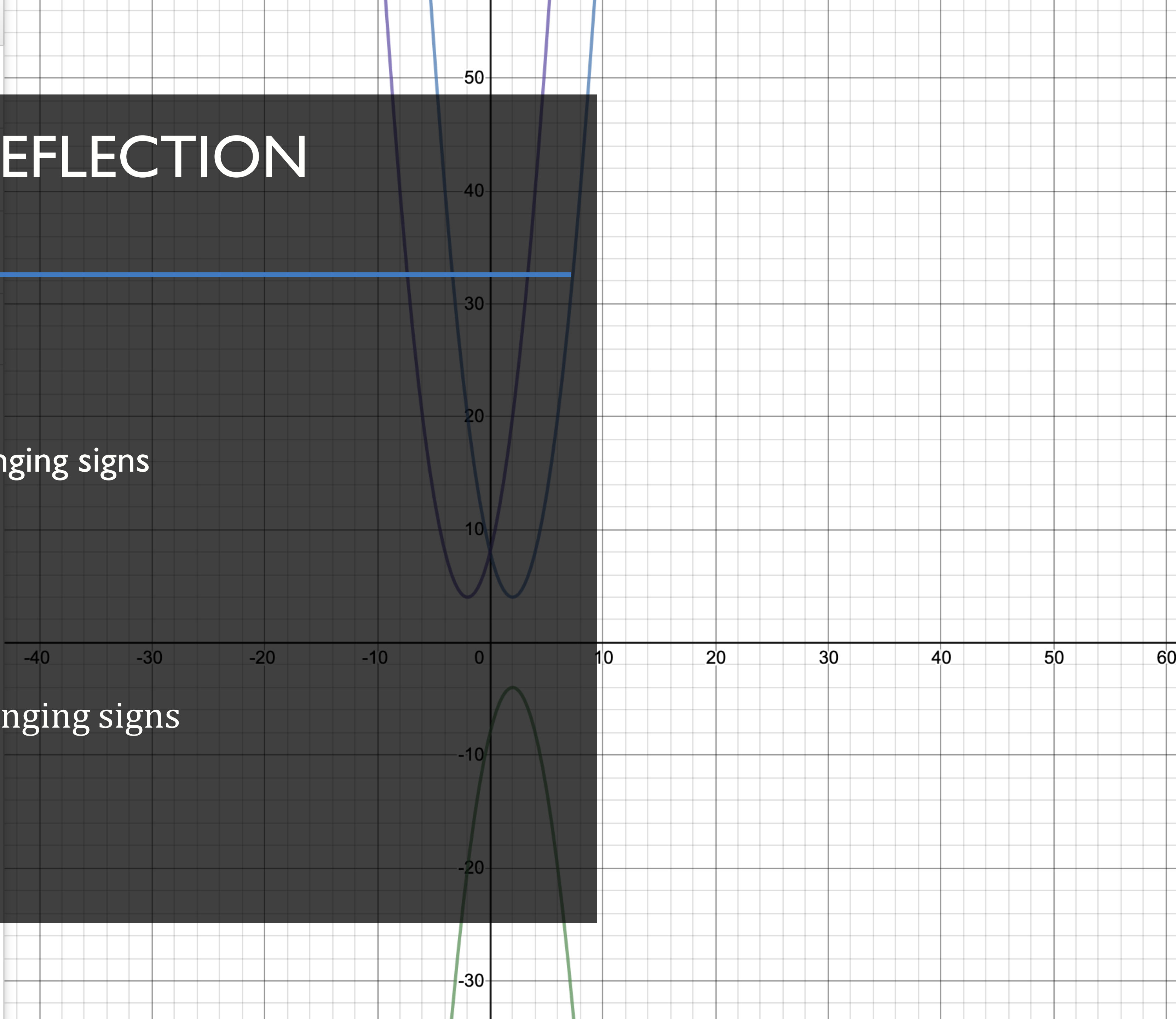
EXAMPLES OF REFLECTION

Reflection over x-axis

- $y = (x-2)^2 + 4$
  - The y values are changing signs
  - $y = -(x-2)^2 - 4$

Reflection over y-axis

- $y = (x-2)^2 + 4$
  - The x values are changing signs
  - $y = (-x-2)^2 + 4$



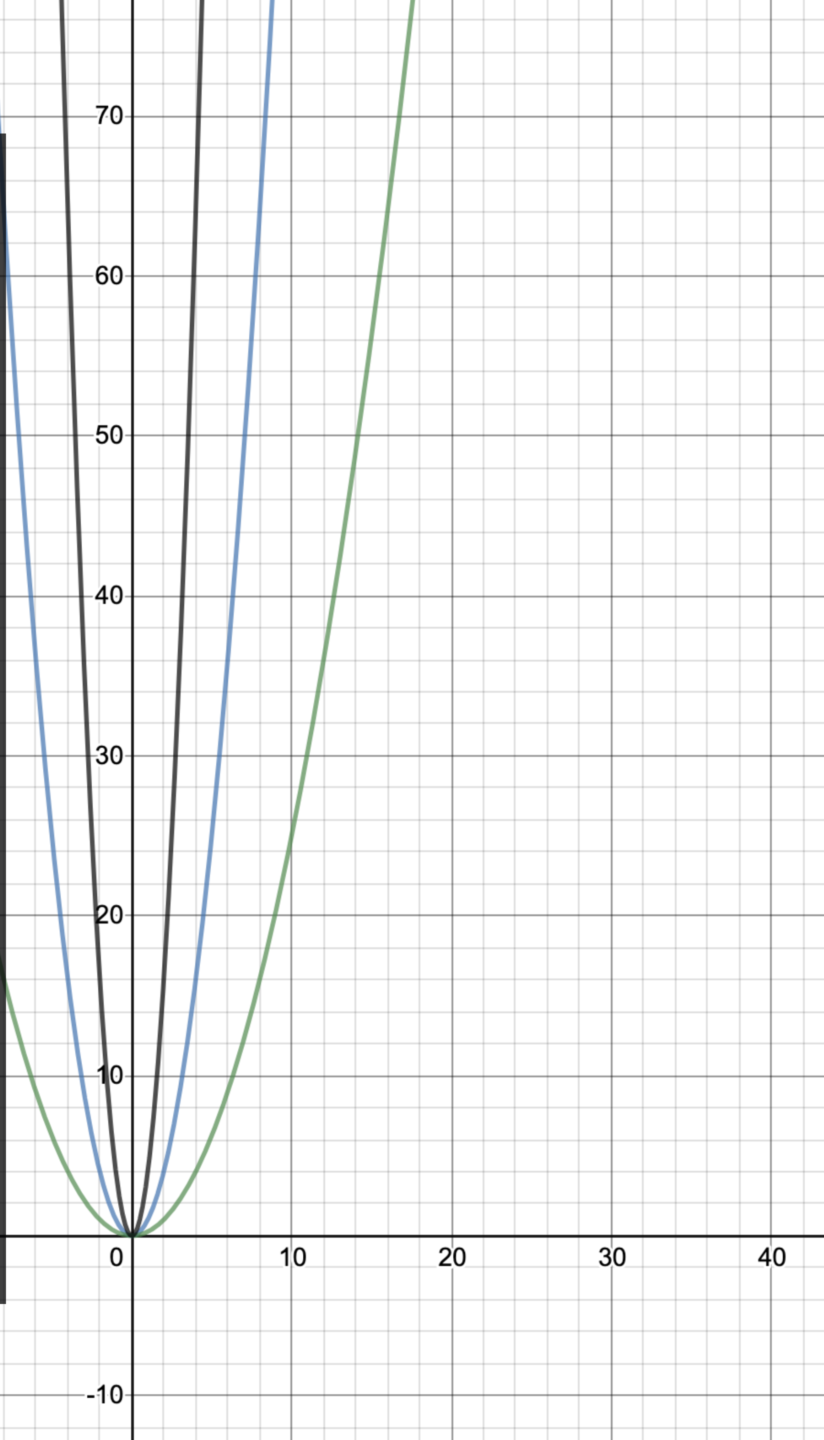
AN EXAMPLE OF DILATION (HORIZONTAL)

Horizontal dilation by a scale factor 2

- $y = x^2$
  - The graph is stretching horizontally by 2
  - $y = \left(\frac{1}{2}x\right)^2 = \frac{1}{4}x^2$

Horizontal dilation by a scale factor ½

- $y = x^2$
  - The graph is shrinking horizontally by ½
  - $y = (2x)^2 = 4x^2$



AN EXAMPLE OF DILATION (VERTICAL)

Vertical dilation by a scale factor 2

- $y = x^2$
  - The graph is stretching vertically by 2
  - $y = 2x^2$

Vertical dilation by a scale factor ½

- $y = x^2$
  - The graph is shrinking vertically by ½
  - $y = \frac{1}{2}x^2$